Direct Search Method to Aeroelastic Tailoring of a Composite Wing under Multiple Constraints

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In order to avoid the possible breakdown of usual optimization methods using gradient information, which is caused by a discontinuous behavior of the flutter velocity as a function of design variables, a feasibility study is made using a direct search method that does not depend on the derivatives of objective/constraint functions. The complex method, as one candidate for such a method, is applied to the minimum weight design of high-aspect-ratio forward/aft swept wings under strength and aeroelastic constraints. It is shown that the complex method is very effective and robust in finding the optimum fiber orientations and the thickness distributions of the upper/lower skin panels of the wing box, especially when the flutter velocity is one of the constraint functions. The deficiency of the complex method is that the rate of convergence rapidly degrades with increasing number of design variables.

Introduction

AEROELASTIC tailoring[1] is a technology to improve the aeroelastic characteristics of a composite wing by positively utilizing the directional stiffness property of composite materials (especially fibrous ones). Since this idea was first proposed by Waddoups et al.[2] in 1971, a considerable amount of theoretical and experimental studies have been accumulated.[1] In order to reflect the idea of aeroelastic tailoring effectively and efficiently in the design of a composite wing, the development of the design tool (a computer program) that utilizes optimization theory is a prerequisite. The function requested of such optimization computer programs is to find the optimum laminate constructions of the upper/lower skin panels that satisfy the multiple design requirements with minimum structural weight. Among such design tools reported so far, the computer program TSO (tailoring and structural optimization program) developed by McCullers and Lynch[3] and FASTOP (flutter and strength optimization program for lifting-surface structures) developed by Wilkinson et al.[4] are notable.

TSO uses the sequential unconstrained minimization technique (SUMT) of Fiacco–McCormick[5] to find the optimum skin-thickness distribution and the fiber orientations, which minimize the objective function under multiple constraints. The objective function may be a combination of weight, lift-curve slope, flexible-to-rigid load ratio, etc., whereas the constraints are minimum and maximum thickness, strength, flutter, and divergence.[6]

FASTOP performs minimum weight design under constraints of strength and flutter. The strength resizing of FASTOP is based on the concept of a fully stressed design, and the procedure employed to realize the structure to meet a minimum flutter-speed requirement is based on the criterion that, for a minimum weight, the derivatives of the flutter speed with respect to element weight must be equal for all elements that were resized to meet the flutter-speed requirement.

It should be noted that, in FASTOP, the fiber-orientation angle of the skin panels are not included in the automated design cycle. That is, the active design variable in the optimization process is the individual ply thickness. In aeroelastic tailoring of a composite wing, it seems to be very important to include the fiber-orientation angle in the automated design process. The effects of the fiber-orientation angle on the aeroelastic characteristics (flutter and divergence) were demonstrated both theoretically[7] and experimentally[8-10] by many researchers. For example, Weisshaar and Foist[7] have shown in their parametric study of the sweptback wing that the flutter and divergence speeds of the wing are very sensitive to the change of the fiber-orientation angle, and the magnitude of the flutter and divergence speed change is very large for a certain range of the fiber-orientation angle. These facts were also confirmed experimentally by the series of experiments by Isogai et al.[8,10] and Ejiri et al.[9] In Ref. 8, it is demonstrated that only the $\pm$ 5-deg change of the fiber orientation of core composite plate of the sweptback wing model produces about 25 ~ 30% difference in flutter velocity in the transonic Mach number range tested. Ejiri et al.[9] have demonstrated by the low-speed wind-tunnel study of a high-aspect-ratio sweptback wing that the large change of the flutter velocity due to the variation of the fiber-orientation angle is associated with the change of the flutter mode; namely, the flutter mode has changed from the mild bending mode predominant flutter at low flutter velocity to the violent torsion mode predominant flutter at high flutter velocity (about 50% higher than that of the low flutter speed).

These facts tell us that, in order to extract the maximum benefit of the aerolastic tailoring, the fiber-orientation angle should be included in the automated design procedure. However, in developing the automated design procedure, in which the fiber-orientation angle is taken as a design variable, we should keep in mind that the flutter mode may change during the design process as demonstrated in the experimental study by Ejiri et al.[9]

In the study of the automated design procedure of a metal wing structure, Haftka[11] has demonstrated that the flutter velocity is discontinuous as a function of the skin thickness, and he has also suggested that this discontinuity may have been caused by the switch between the two flutter modes. It seems to be quite possible that, for a composite wing, the flutter velocity may be discontinuous as a function of the fiber-orientation angle also. This means that the optimization procedures, which utilize the flutter-velocity derivative with respect to the design variables such as fiber-orientation angle/skin

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thickness, may break down at the point where the flutter velocity changes discontinuously.

As already described, both TSO and FASTOP use flutter-velocity derivatives in their optimization processes. As far as we know, no optimization procedure for aeroelastic tailoring, which does not depend on the flutter-velocity derivatives, has yet been published.

In view of this, we have developed a preliminary design code that does not depend on the derivatives of objective and constraint functions. The method we have employed is the complex method originally proposed by Box,[12] which can handle multiple constraints without recource to gradients. In this paper, we will describe the details of the procedure of our preliminary design code and some of the numerical results obtained in the preliminary design study of high-aspect-ratio forward/aft swept composite wings.

Application of the Complex Method to Aeroelastic Tailoring of a Simplified Wing-Box Structure

Since the detailed description about the complex method is given in Ref. 13, we will explain here its application to the optimization of the laminate construction of the upper/lower skin panels of a wing-box structure. To simplify the analysis, the wing-box structure is assumed to be composed of the upper and lower skin panels and the front and rear spars, as shown in Fig. 1. It is also assumed that the upper and lower skin panels can be treated as membranes and that the front and rear spars can be treated as shear webs.

As to the laminate construction of the membrane (either upper or lower), the following assumptions are made: namely, it is symmetrical (about the skin's own midsurface) and is composed of $2n$ layers, each layer of which has the same thickness and different fiber-orientation angle θ_i $(i = 1, \ldots, n)$ (see Fig. 2). The total thickness t of the membrane is assumed to be constant in the chordwise direction but the spanwise distribution is assumed to be expressed by a polynomial of $(m\text{-}1)$th degree, also shown in Fig. 2. Thus, as the design variables, we can select the membrane thickness $t_1, t_2, \ldots, t_m$ at m spanwise stations (varying from the root station to the tip station) and each fiber orientation $\theta_1, \theta_2, \ldots, \theta_n$ of n layers. Therefore, we have a total of $(m+n)$ design variables.

The objective function is the structural weight of the wing box, namely, the total weight of upper and lower skin panels and the front and rear spars. As to the front and rear spars, however, the thickness distribution and laminate constructions are not included in the design variables; that is, the front and rear spars are assumed to be the linearly tapered shear webs with a quasi-isotropic laminate construction.

Thus, if we denote the design variables as x_i $(i = 1, 2, \ldots, m+n)$ in applying the complex method, we can write

$$x_i = C_t t_i, \qquad i = 1, \ldots, m \tag{1a}$$

$$= C_\theta \theta_{i-m}, \qquad i = m+1, \ldots, m+n \tag{1b}$$

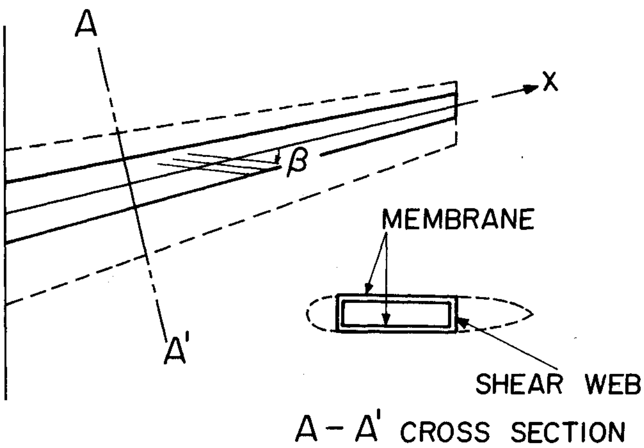

Fig. 1 Simplified model of wing-box structure.

where C_t and C_θ are constant scale factors. The values we have used for C_t and C_θ are $C_t = 1000$ and $C_\theta = 1$, which produce the same order of variations of x_i when t_i varies about 0.001 m and when θ_i varies about 1 deg. (The units used for t_i and θ_i are meter and degree, respectively.)

The (inequality) constraints we have considered so far are static strength under given load and aeroelastic constraints where flutter velocity, divergence velocity, and static aeroelastic deformation (vertical deflection and twist angle at the wing tip station) are included. In addition to these constraint functions, we impose some constraints on the skin-thickness distributions, namely,

$$t_i \geq t_{min}, \qquad t_1 \geq t_2 \geq t_3 \geq \ldots \geq t_m \tag{2}$$

where t_{min} is the minimum gage and the latter condition is imposed to avoid the stituation that the solution gets into the impractical local minimum.

The analytical method used for the strength and vibration analyses of the simplified box beam shown in Fig. 1 is the beam theory, which is similar to that described in Ref. 14. The failure criterion used in the present strength analysis is that of Tsai-Wu.[15] That is, the strength ratio R of the weakest layer, which is the ratio between the allowable strength and the applied stress, is evaluated to identify the failure of the box beam. For the static and dynamic aeroelastic analyses, the conventional Rayleigh-Ritz method is employed, in which the normal vibration modes and the steady/unsteady aerodynamic forces predicted by the subsonic lifting-surface theory (doublet lattice method[16]) are utilized.

In the complex method, the optimization process proceeds by the use of a flexible figure of $k \geq (n+m+1)$ vertices, which can expand or contract in any or all directions, handling

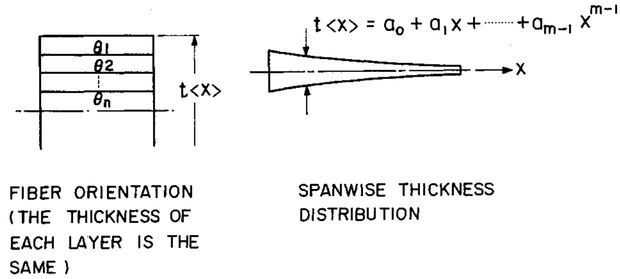

Fig. 2 Laminate constructions of upper/lower skin panels.

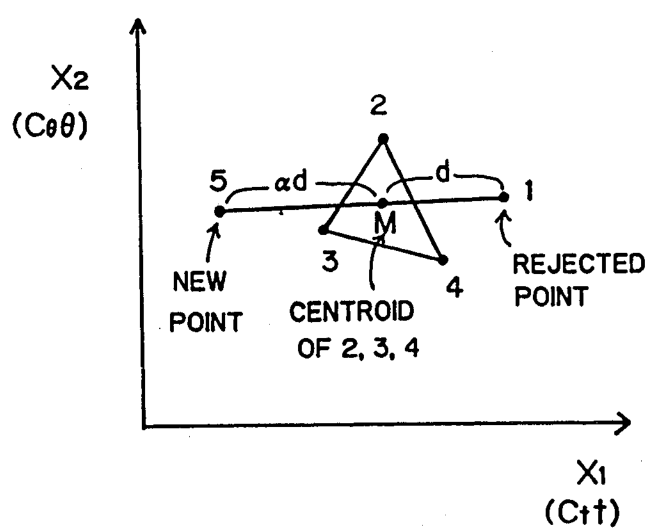

Fig. 3 Procedure of the complex method.

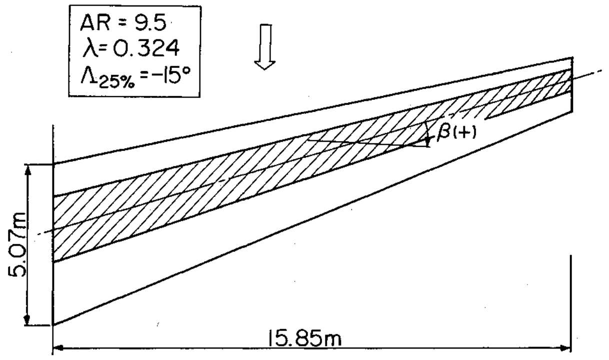

Fig. 4a Planform of high-aspect-ratio forward swept wing.

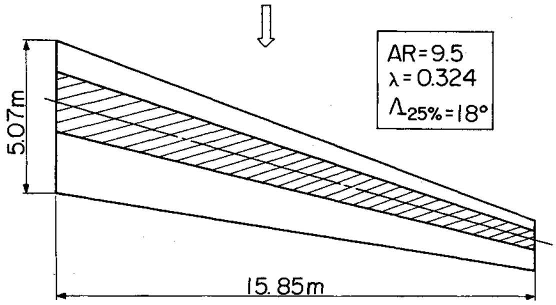

Fig. 4b Planform of high-aspect-ratio aft swept wing.

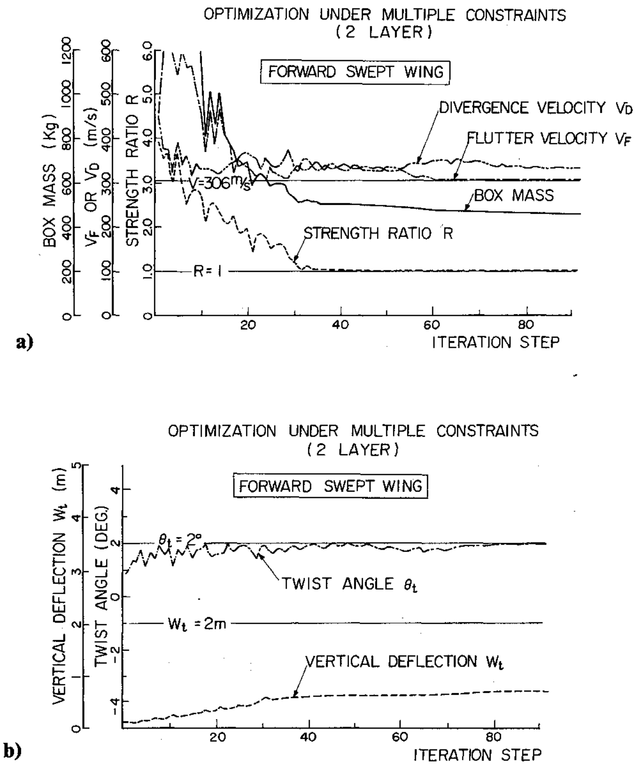

Fig. 5 Variations of box mass, strength ratio, and aeroelastic characteristics during optimization process.

constraints in an appropriate manner. The procedure of the complex method is best illustrated by a two-dimensional problem, as shown in Fig. 3. In this illustrative example, the skin is the unidirectional laminate of uniform thickness t and fiber orientation θ. Therefore, the design variables are $x_1 = C_t t$ and $x_2 = C_\theta \theta$. The points 1–4 are the initial vertices. The objective function (box mass) and the constraint functions (flutter and divergence speeds, static deflections, strength ratio, etc.) at each design point are first evaluated. Then, the box masses at these design points are compared and the worst point (point 1) is to be rejected. The centroid of points 2–4 is designated as M. If the distance between the point 1 and the centroid M is d, then the point 1 will be replaced by a point 5, which is a distance αd from M in a direction defined by the line drawn from 1 through M. The coordinates of point 5 are then given by

$$x_{i,5} = \alpha(x_{i,M} - x_{i,1}) + x_{i,M}, \qquad i = 1, 2 \tag{3}$$

where the centroid M is given by

$$x_{i,M} = \frac{1}{3}\sum_{j=2}^{4} x_{i,j}, \qquad i = 1, 2 \tag{4}$$

Then, the objective and constraint functions at the new point 5 are evaluated. If the point 5 is a feasible point, the process of vertex rejection and regeneration is repeated. If, however, any of the constraints are violated, the new vertex is moved halfway in toward the centroid. If this constraint is still violated, this retreat is continued until a valid point is attained. The process is repeated by applying the procedures until the value of the box mass converges. Thus, we only need to evaluate the objective and constraint functions at the new vertex at each iteration step. The algorithm is very simple and easy to program, and there is no need to evaluate the gradients of the objective and constraint functions. Further details of the procedure are explained in Ref. 13.

In our preliminary design code, we can handle the number of laminates (of upper/lower skin panels) up to four layers, while the thickness distributions of the panels are assumed to be expressed by a polynomial of two degree. Therefore, the number of design variables that can be handled are up to seven. As to the number of vertices k used in the complex method, we have so far obtained a good performance by using k = (the number of design variables + 3). Although we use k vertices, the initial vertices can be found by specifying only one vertex that satisfies all of the constraints and by distributing the remaining $k-1$ vertices randomly around the first vertex. (See further details of this procedure in Ref. 13.)

Numerical Examples

In order to evaluate the preliminary design code just described, we have made the design studies of high-aspect-ratio forward/aft swept wings, which are the main wings of a 150 seater. The planforms of the wings are shown in Figs. 4a and 4b, respectively. The aspect ratio of the wings is the same 9.5. The swept angle of the quarter-chord line is −15 deg for the forward swept wing, and 18 deg for the aft swept wing. (The forward and aft swept angles of the wing box are the same 16.5 deg for both wings.) The taper ratio of both wings is the same 0.324. Both wings have the natural laminar flow-type supercritical airfoil sections of about 12% thick. The hatched parts of the wings show the location of the box beams. That is, the front and rear spars are located along the 20 and 60% chord lines, respectively. On these wings, we tentatively impose the following static strength and aeroelastic requirements:

1) It should sustain the static aerodynamic loading induced by 3.8g flight condition, which corresponds with the loading of about 1200 kN acting on the semispan wing. (The load distributions induced at $M = 0.75$ and $\alpha = 2$ deg are assumed for both wings.)

2) The flutter/divergence velocity should clear 1.2 V_{max} (= 306 m/s) (V_{max} is the design diving speed) at $M = 0.75$ at the altitude of 12,500 ft.

3) The vertical deflection and twist angle (positive tail-down) at the tip station under 1g flight condition should be less than 2 m and 2 deg, respectively.

Under these multiple constraints, the minimum weight design to find the optimum laminate constructions was performed. The material used for the design is graphite-epoxy (T300/5208). The thickness of the front and rear shear webs are assumed to be 12 mm at the root station and to be tapered linearly to the tip station with taper ratio 0.324.

Results for Forward Swept Wing

In Figs. 5a and 5b, the results obtained for the case where the upper/lower skin panels of the box structure are composed of two layers are shown. In the figures, the variations of the box mass, the strength ratio R of the weakest layer, the flutter and divergence velocities, and the vertical deflection W_t and the twist angle θ_t of the tip station during the optimization process are plotted, respectively. As seen from Figs. 5, the convergence was obtained after about 80 iteration steps. The strength ratio R, the flutter velocity, and the twist angle of the tip station reach the value of each constraint, respectively, while the divergence velocity and the vertical deflection of the tip station have some room to the constraints. The converged value of box mass (the mass of the upper/lower skin panels and front/rear spars) is 461 kg. The laminate constructions obtained are $t_1 = 8.07$ mm, $t_2 = 4.23$ mm, $t_3 = 1.08$ mm, $\theta_1 = 7.89$ deg, and $\theta_2 = -19.98$ deg, respectively, where t_1, t_2, and t_3 are the thicknesses of the skin panels at the root, midsemispan, and tip station, respectively.

The flutter and divergence velocities at the optimum point are $V_F = 306$ m/s and $V_D = 336$ m/s, respectively. The strength ratio is $R = 1.0$, and the static aeroelastic deflection under 1g condition is $W_t = 0.72$ m and $\theta_t = 2.0$ deg.

In order to see the amount of the weight reduction attained by the optimization, we have also designed the wing box, the laminate constructions of whose skin panels are conventional (0 deg: 33.3%; 45 deg: 33.3%; −45 deg: 33.3%)s. The thickness of the upper/lower skin panel is determined by trial and error, so that the same design requirements as those for the optimized one are satisfied. The t_1, t_2, and t_3 of each panel thus determined are 19.2, 10.1, and 2.57 mm, respectively. The total mass of the wing box becomes 877 kg, which corresponds with 461 kg of the optimized one. Therefore, the weight reduction attained by the optimization is about 47%.

The detailed results obtained for the optimized (case A) and nonoptimized (case D) wings, which were just described, are shown in Table 1. In order to see the effects of the constraints on the weight of the wing box, the two additional cases are also studied, that is, case B where the aeroelastic constraints are dropped (only the static strength constraint is satisfied) and case C where the static strength constraint is dropped (only the aeroelastic constraints are satisfied). The detailed results of cases B and C are also shown in Table 1. As seen in Table 1, the weight reduction of case B is very large but all of the aeroelastic requirements are not satisfied. In case C, the static strength requirement is not satisfied, although the aeroelastic requirements are satisfied. These facts suggest that, in aeroelastic tailoring, it is extremely important to impose both strength and aeroelastic requirements in the optimization process.

The results described previously are for the cases for which the upper/lower skin panels of the wing box are composed of two layers. We have also tried the cases of three and four layers, respectively, under the same design requirements, but no further improvement of the weight reduction over the two-layer case was obtained. It is also observed that the convergence of the optimization process is degraded rapidly with increasing number of design variables.

Results for Aft Swept Wing

In Figs. 6a and 6b, the results obtained for the aft swept wing, whose box skin is composed of two layers, are shown. As already described previously, the strength and aeroelastic requirements for this aft swept wing are the same as those for the previous forward swept wing. As seen in the figures, both flutter velocity and strength ratio reach the value of each constraint, while the divergence velocity and the static aeroelastic deflections at the tip station have some room to their constraints. The converged values of the wing box mass is 447 kg. The laminate constructions obtained are $t_1 = 8.44$ mm, $t_2 = 3.82$ mm, and $t_3 = 0.79$ mm, and $\theta_1 = 15.37$ deg and $\theta_2 = -9.07$ deg. Further details of the strength and aeroelastic characteristics obtained at the optimum point is shown in Table 2.

In order to see the amount of weight reduction attained by the optimization, we have also designed the wing box, the

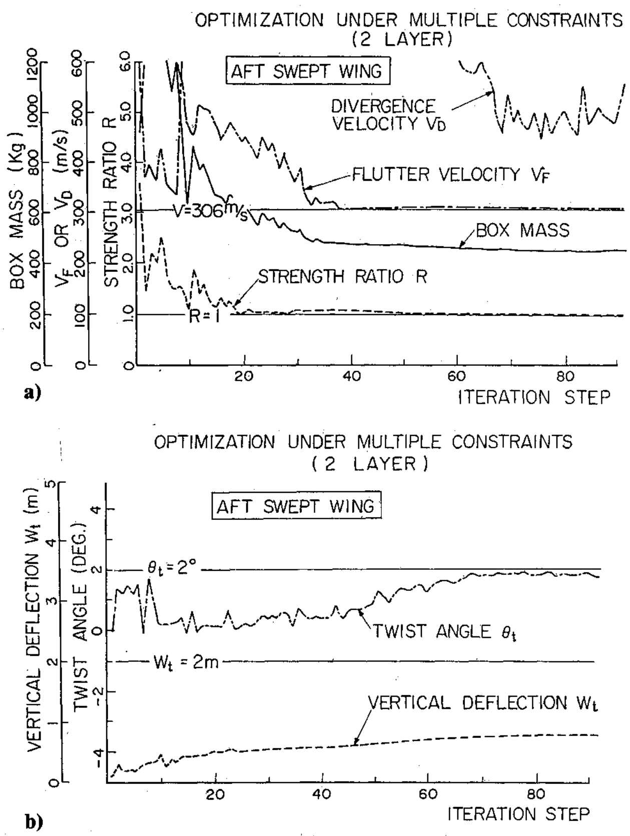

Fig. 6 Variations of box mass, strength ratio, and aeroelastic characteristics during optimization process.

Table 1 Effect of optimization on structural weight of wing box of high-aspect-ratio forward swept wing[a]

	Case	Constraints	Box mass, kg	Strength ratio	V_F, m/s	V_D, m/s	W_t, m	θ_t, deg
Optimized	A	Strength and aeroelasticity	461	1.01	306	336	0.72	1.98
	B	Strength only	362	1.00	146	221	1.23	7.46
	C	Aeroelasticity only	408	0.49	306	326	1.07	1.98
Nonoptimized	D	Strength and aeroelasticity	877	1.00	656	346	0.57	1.57

[a]Laminate construction of case A: $t_1 = 8.1$ mm, $t_2 = 4.2$ mm, $t_3 = 1.1$ mm, $\theta_1 = 7.9$ deg, and $\theta_2 = -20.0$ deg.
Laminate construction of case D: $t_1 = 19.2$ mm, $t_2 = 10.1$ mm, $t_3 = 2.6$ mm, $\theta_1 = 0$ deg, $\theta_2 = 45$ deg, and $\theta_3 = -45$ deg.
Weight reduction = 47%.

Table 2 Effect of optimization on structural weight of wing box of high-aspect-ratio aft swept wing[a]

	Case	Constraints	Box mass, kg	Strength ratio	V_F, m/s	V_D, m/s	W_t, m	θ_t, deg
Optimized	A	Strength and aeroelasticity	447	1.0	306	542	0.81	1.77
Nonoptimized	D	Strength and aeroelasticity	846	1.0	584	∞	0.51	−0.62

[a]Laminate construction of case A: $t_1 = 8.4$ mm, $t_2 = 3.8$ mm, $t_3 = 0.8$ mm, $\theta_1 = 15.4$ deg, and $\theta_2 = -9.1$ deg.
Laminate construction of case D: $t_1 = 20.2$ mm, $t_2 = 9.1$ mm, $t_3 = 1.9$ mm, $\theta_1 = 0$ deg, $\theta_2 = 45$ deg, and $\theta_3 = 45$ deg.
Weight reduction = 47%.

laminate constructions of whose skin panels are conventional (0 deg: 33.3%; 45 deg: 33.3%; −45 deg: 33.3%)s. The thickness of the upper/lower skin panels is determined by trial and error, so that the same design requirements as those for the optimized one are satisfied. The t_1, t_2, and t_3 of each panel thus determined are 20.2, 9.1, and 1.9 mm, respectively. Further details of the strength and aeroelastic characteristics of this nonoptimized wing are shown in Table 2 also. The strength ratio R of the weakest layer is 1.0, while the flutter velocity is 583 m/s, which is far above the design requirement of 306 m/s. The total mass of the wing box becomes 846 kg, which corresponds with 447 kg of the optimized one. Therefore, the weight reduction attained by the optimization is about 47%.

It is of great interest to see that the weight reductions attained by the optimization are almost the same for both forward swept and aft swept wings considered here.

Concluding Remarks

The preliminary design program, which performs the minimum weight design under the static strength and aeroelastic constraints, was described. Both the thickness distribution and the fiber-orientation angle of the upper/lower skin panels of the simplified wing box are taken as the active design variables of the automated design procedure. In order to avoid the possible breakdown of the usual optimization method using the gradient information, which is caused by the discontinuous behavior of the flutter velocity, the complex method of Box, which is one of the direct search methods that does not depend on the derivatives of objective/constraint functions, is employed. The performance of the code is demonstrated by applying it to the designs of the high-aspect-ratio forward/aft swept wings. It is shown that about 47% of weight reduction of box structure is obtained (both for the forward/aft swept wings considered) by the optimization compared with those of the nonoptimized designs.

The complex method is very effective and robust when the fiber-orientation angle is taken as one of the design variables and when the flutter velocity is one of the constraints. The deficiency of the complex method is that the rate of convergence is rapidly degraded with increasing number of design variables.

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